Howework 5 – Computer Architecture

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1 Question 1

1.1 Sub-question 1

Starting with: $f(A,B,C,D) = \overline{A} \times \overline{B} \times \overline{C} \times \overline{D} + \overline{A} \times \overline{B} \times C \times \overline{D} + \overline{A} \times B \times C \times \overline{D} + A \times \overline{B} \times \overline{C} \times \overline{D} + A \times \overline{B} \times \overline{C} \times D + A \times \overline{B} \times C \times \overline{D} + A \times B \times C \times D + \overline{A} \times \overline{B} \times \overline{C} \times D + \overline{A} \times B \times C \times D =$

apply the distributive law: $\overline{A} \times (\overline{B} \times \overline{C} \times \overline{D} + \overline{B} \times C \times \overline{D} + B \times C \times \overline{D} + \overline{B} \times \overline{C} \times D + B \times C \times D) + A \times (\overline{B} \times \overline{C} \times \overline{D} + \overline{B} \times \overline{C} \times D + \overline{B} \times C \times \overline{D} + B \times C \times D) =$

apply it again: $\overline{A} \times (\overline{B} \times (\overline{C} \times \overline{D} + C \times \overline{D} + \overline{C} \times D) + B \times C \times (\overline{D} + D)) + A \times (\overline{B} \times (\overline{C} \times \overline{D} + \overline{C} \times D + C \times \overline{D}) + B \times C \times D) =$

1.1.1 Solving $\overline{C} \times \overline{D} + C \times \overline{D} + \overline{C} \times D$

Apply the distributive law: $\overline{C} \times (\overline{D} + D) + C \times \overline{D}$ =i

then apply the inverse law: $\overline{C} \times 1 + C \times \overline{D} =$

then apply the identity law: $\overline{C} + C \times \overline{D} =$

then apply De Morgan's law: $\overline{C \times \overline{C \times \overline{D}}} =$

then apply it again: $\overline{C \times (\overline{C} + D)} =$

then apply the distributive law: $\overline{C \times \overline{C} + C \times D} =$

then apply the inverse law: $\overline{0 + C \times D} =$

then, finally, apply the identity law, obtaining: $\overline{C \times D}$

1.1.2 Back to the main function

by 1.1.1 and the inverse law, we continue this way: $\overline{A} \times (\overline{B} \times \overline{C \times D} + B \times C \times 1) + A \times (\overline{B} \times \overline{C \times D}) + B \times C \times D) =$

then apply the identity law: $\overline{A} \times (\overline{B} \times \overline{C \times D} + B \times C) + A \times (\overline{B} \times \overline{C \times D}) + B \times C \times D) =$

then apply the distributive law: $\overline{A} \times \overline{B} \times \overline{C \times D}) + \overline{A} \times B \times C + A \times \overline{B} \times \overline{C \times D} + A \times B \times C \times D =$

then apply it again: $\overline{B} \times \overline{C \times D} \times (\overline{A} + A) + B \times C \times (\overline{A} + A \times D) =$

then apply the inverse law: $\overline{B} \times \overline{C \times D} \times 1 + B \times C \times (\overline{A} + A \times D) =$

then apply the identity law: $\overline{B} \times \overline{C \times D} + B \times C \times (\overline{A} + A \times D) =$

then apply De Morgan's law: $\overline{B} \times \overline{C \times D} + B \times C \times \overline{A \times \overline{A \times D}} =$

then apply it again: $\overline{B} \times \overline{C \times D} + B \times C \times \overline{A \times (\overline{A} + \overline{D})} =$

then apply the distributive law: $\overline{B} \times \overline{C \times D} + B \times C \times \overline{A \times \overline{A} + A \times \overline{\overline{D}}} =$

then apply the inverse law: $\overline{B} \times \overline{C \times D} + B \times C \times \overline{0 + A \times \overline{\overline{D}}} =$

then apply the identity law: $\overline{B} \times \overline{C \times D} + B \times C \times \overline{A \times \overline{\overline{D}}} =$

then, finally, apply De Morgan's law, we find the result: $\overline{B} \times \overline{C \times D} + B \times C \times (\overline{A} + D)$

1.2 Sub-question 2

Using the minterm normal form given in section 1.1, we can deduct the truth table of the function. Using that, we can find the following Karnaugh map and the following prime implicants:

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	1	1	0
10	1	1	0	1

Using those prime implicants we find the boolean function $(\overline{B} + C) \times (\overline{A} + \overline{B} + D) \times (B + \overline{C} + \overline{D})$.

2 Question 2

2.1 Sub-question 1

We assume X_3 is the most significant bit in the binary number. The truth table of the function is:

X_3	X_2	X_1	X_0	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

2.2 Sub-question 2

The Conjunctive Normal Form, or the maxterm expansion of the function, is:

$$(X_3 + X_2 + X_1 + X_0) \times (X_3 + X_2 + \overline{X_1} + X_0) \times (X_3 + X_2 + \overline{X_1} + \overline{X_0}) \times (X_3 + \overline{X_2} + X_1 + X_0) \times (X_3 + \overline{X_2} + X_1 + \overline{X_0})$$

The Disjunctive Normal Form, or the minterm expansion of the function, is:

$$(\overline{X_3} \times \overline{X_2} \times \overline{X_1} \times X_0) + (\overline{X_3} \times X_2 \times X_1 \times \overline{X_0}) + (\overline{X_3} \times X_2 \times X_1 \times X_0) + (X_3 \times \overline{X_2} \times \overline{X_1} \times \overline{X_0}) + (X_3 \times \overline{X_2} \times \overline{X_1} \times X_0) + (X_3 \times \overline{X_2} \times X_1 \times \overline{X_0}) + (X_3 \times \overline{X_2} \times X_1 \times X_0) + (X_3 \times X_2 \times \overline{X_1} \times \overline{X_0}) + (X_3 \times X_2 \times \overline{X_1} \times X_0) + (X_3 \times X_2 \times X_1 \times \overline{X_0}) + (X_3 \times X_2 \times X_1 \times X_0)$$

2.3 Sub-question 3

The maxterm expansion of the function above is clearly the best approach between the two, since it contains fewer terms (and thus requires less logic gates).

2.4 Sub-question 4

Please find the Logisim file answering this question named `CA_2.4_Homework_5.circ` in the zip file.

2.5 Sub-question 5

Please find the Logisim file answering this question named `CA_2.5_Homework_5.circ` in the zip file.

3 Question 3

The bomb will not explode if either the first, the second or the fourth cable from the left are cut. For the first and the second cable this happens because the NOR inside the circuit will get at least a 1 as input, and therefore it will produce a 0 as output, pulling the final AND output to 0. When the fourth cable is cut, the NOT will give always 0 as output and therefore the final AND output will be always 0.