Computer Architecture – Assignment 11 Bonus

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1 Bonus Question 1

1.1 Decimal to Hexadecimal

All the numbers are going to be first converted to binary and then to hexadecimal for an easier calculation

a) **11**;

First we find how 11 is expressed in binary which is 1011 and then we convert it to scientific base 2 notation: $1011 = 1.\mathbf{011} \times 2^3$.

The sign bit is 0 since $11 > 0$ and therefore the number is positive.

To determine the exponent we take the single precision bias (127) and we add to it the amount of times we moved a bit past the floating point: $127 + 3 = 130$.

We then convert 130 to decimal by successive halving:

Fraction	**Rest**
$130/2 = 65$	0
$65/2 = 32$	1
$32/2 = 16$	0
$16/2 = 8$	0
$8/2 = 4$	0
$4/2 = 2$	0
$2/2 = 1$	0
$1/2 = 0$	1

By reading from most significant bit to least significant bit we get 10000010 for our exponent.

We then finally assemble the floating point binary number with the parts we have converted:

0 10000010 01100000000000000000000 = 01000001001100000000000000000000

And we convert to hexadecimal by splitting the number in 4 bit groups and converting each group to its equivalent hexadecimal digit:

0100 0001 0011 0000 0000 0000 0000 0000 = 4 1 3 0 0 0 0 0 = 41300000.

From now on the passages are going to include only the calculations to avoid verbosity. Nonetheless, the same reasoning will be applied for each conversion.

b) **5/64**;

Sign bit = 0

$\frac{5}{64} = 0.078125$

Conversion to binary:

- $0.078125 \times 2 = \mathbf{0} + 0.15625$;
- $0.15625 \times 2 = \mathbf{0} + 0.3125$;
- $0.3125 \times 2 = \mathbf{0} + 0.625$;
- $0.625 \times 2 = \mathbf{1} + 0.25$;
- $0.25 \times 2 = \mathbf{0} + 0.5$;
- $0.5 \times 2 = \mathbf{1} + 0$;

Result of conversion: 0.000101.

Normalization to scientific notation: $1.\mathbf{01} \times 2^{-4}$.

Exponent bits in decimal = $127 - 4 = 123$

Conversion of exponent:

Fraction	**Rest**
123/2 = 61	1
61/2 = 30	1
30/2 = 15	0
15/2 = 7	1
7/2 = 3	1
3/2 = 1	1
1/2 = 0	1

Exponent bits in binary = 01111011

Final binary number = 0 01111011 01000000000000000000000 =

00111101101000000000000000000000

Conversion to hexadecimal:

0011 1101 1010 0000 0000 0000 0000 0000 = 3 D A 0 0 0 0 0 = 3DA00000.

c) **-5/64**;

With this being the negative counterpart of the previous number we just need to flip the sign bit of $\frac{5}{64}$.

With that we get: 10111101101000000000000000000000.

Which, converted to hexadecimal, gives:

1011 1101 1010 0000 0000 0000 0000 0000 = B D A 0 0 0 0 0 = BDA00000.

d) **6.125**;

Sign bit = 0

Conversion to binary:

Integer part = $6_{10} = 110_2$

Fractional part = 0.125_{10}:

- $0.125 \times 2 = \mathbf{0} + 0.25$;
- $0.25 \times 2 = \mathbf{0} + 0.5$;

- $0.5 \times 2 = \mathbf{1} + 0$;

$= 001_2$.

Normalized binary $= 110.001 = 1.\mathbf{10001} \times 2^2$

Exponent bits in decimal $= 127 + 2 = 129$.

Fraction	**Rest**
$129/2 = 64$	1
$64/2 = 32$	0
$32/2 = 16$	0
$16/2 = 8$	0
$8/2 = 4$	0
$4/2 = 2$	0
$2/2 = 1$	0
$1/2 = 0$	1

Exponent bits in binary = 10000001

Final binary number = 0 10000001 10001000000000000000000 =

01000000110001000000000000000000.

Conversion to hexadecimal: 0100 0000 1100 0100 0000 0000 0000 0000 =
4 0 C 4 0 0 0 0 = 40C40000.

1.2 Hexadecimal to Decimal

All the numbers are going to be first converted to binary and then to decimal for an easier calculation

a) **42E48000**$_{16}$

Conversion to binary:

42E48000 = 0100 0010 1110 0100 1000 0000 0000 0000 =

0 10000101 11001001000000000000000 =

01000010111001001000000000000000

From this we can observe that the sign bit is 0 (i.e. positive).

Conversion of exponent from binary to decimal:

$10000101 = 2^7 + 2^2 + 2^0 = 128 + 4 + 1 = 133$

To find how many bits have been moved beyond the floating point we now find the exponent for our notation:

$133 - 127 = 6$

And we then take the mantissa to multiply it by 2^6 (where 6 is the number we found through the last passage) to get the final binary number to convert to decimal:

$1.11001001000000000000000 \times 2^6 = 1.11001001 \times 2^6 = 1110010.01$

Conversion to decimal:

$1110010.01 = 2^6 + 2^5 + 2^4 + 2 + 2^{-2} = 64 + 32 + 16 + 2 + 0.25 = 114.25$

b) **3F880000**$_{16}$

Conversion to binary:

$3F880000 = 0011\ 1111\ 1000\ 1000\ 0000\ 0000\ 0000\ 0000\ =$

$0\ 01111111\ 00010000000000000000000 =$

$00111111100010000000000000000000$

Sign bit $= 0$

Conversion of exponent from binary to decimal:

$01111111 = 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2 + 1 = 64 + 32 + 16 + 8 + 4 + 2 + 1 = 127$

Power of $2 = 127 - 127 = 0$

Final binary number:

$1.0001 \times 2^0 = 1.0001$

Conversion to decimal:

$1.0001 = 2^0 + 2^{-4} = 1 + 0.0625 = 1.0625$

c) **00800000**$_{16}$

Conversion to binary:

$00800000 = 0000\ 0000\ 1000\ 0000\ 0000\ 0000\ 0000\ 0000 =$

$0\ 00000001\ 000\ 00000000000000000000 =$

$00000000100000000000000000000000$

Sign bit $= 0$

Conversion of exponent from binary to decimal:

$00000001 = 2^0 = 1$

Power of $2 = 1 - 127 = -126$

Final binary number:

$1.00000000000000000000 \times 2^{-126}$

Conversion to decimal:

$1.00000000000000000000 \times 2^{-126} = 2^{-126} = 1.1754944 \times 10^{-38}$

d) **C7F00000**$_{16}$

Conversion to binary:

$C7F00000 = 1100\ 0111\ 1111\ 0000\ 0000\ 0000\ 0000\ 0000 =$

$1\ 10001111\ 11100000000000000000000 =$

$11000111111100000000000000000000$

Sign bit $= 1$

Conversion of exponent from binary to decimal:

$10001111 = 2^7 + 2^3 + 2^2 + 2^1 + 2^0 = 128 + 8 + 4 + 2 + 1 = 143$

Power of $2 = 143 - 127 = 16$

Final binary number:

$1.11100000000000000000000 \times 2^{16} = 1.111 \times 2^{1}6 =$

11110000000000000.0

Conversion to decimal:

$11110000000000000.0 = 2^{1}6+2^{1}5+2^{1}4+2^{1}3 = 65536+32768+16384+8192 = 122880$ and, since the sign bit is 1 (i.e. negative), $= -122880$.

2 Bonus Question 2