

# Howework 3 – Introduction to Computational Science

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## Question 1

$$i = 1 \quad l_1 = \begin{bmatrix} 1 \\ 1 \\ 5 \\ -3 \end{bmatrix} \quad u_1 = [2 \quad 1 \quad 1 \quad -2]$$

$$A_2 = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 2 & 2 & -2 & -1 \\ 10 & 4 & 23 & -8 \\ -6 & -2 & 4 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 1 & -2 \\ 2 & 1 & 1 & -2 \\ 10 & 5 & 5 & -10 \\ -6 & -3 & -3 & 6 \end{bmatrix} = \begin{bmatrix} & & & \\ & 1 & -3 & 1 \\ & -1 & 18 & 2 \\ & 1 & 7 & 0 \end{bmatrix}$$

$$i = 2 \quad l_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad u_2 = [ \quad 1 \quad -3 \quad 1 ]$$

$$A_3 = \begin{bmatrix} & 1 & -3 & 1 \\ & -1 & 18 & 2 \\ & 1 & 7 & 0 \end{bmatrix} - \begin{bmatrix} & 1 & -3 & 1 \\ & -1 & 3 & -1 \\ & 1 & -3 & 1 \end{bmatrix} = \begin{bmatrix} & & & \\ & 15 & 3 & \\ & 10 & -1 & \end{bmatrix}$$

$$i = 3 \quad l_3 = \begin{bmatrix} 1 \\ 2/3 \end{bmatrix} \quad u_3 = [ \quad 15 \quad 3 ]$$

$$A_4 = \begin{bmatrix} & & & \\ & 15 & 3 & \\ & 10 & -1 & \end{bmatrix} - \begin{bmatrix} & 15 & 3 & \\ & 10 & 2 & \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & -3 \end{bmatrix}$$

$$i = 4 \quad l_4 = \begin{bmatrix} \\ \\ 1 \end{bmatrix} \quad u_4 = [ \quad \quad -3 ]$$

$$L = \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ 5 & -1 & 1 & \\ -3 & 1 & 2/3 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 & 1 & -2 \\ & 1 & -3 & 1 \\ & & 15 & 3 \\ & & & -3 \end{bmatrix}$$

$$Ly = B \Rightarrow \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ 5 & -1 & 1 & \\ -3 & 1 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 36 \\ 18 \end{bmatrix}$$

$$y_1 = -1$$

$$y_2 = -3 - (-1) \cdot 1 = -2$$

$$y_3 = 36 - 1 \cdot 2 - (-1) \cdot 5 = 39$$

$$y_4 = 18 - \frac{2}{3} \cdot 39 - (-2) - (-3) \cdot (-1) = -9$$

$$Ux = Y \Rightarrow \begin{bmatrix} 2 & 1 & 1 & -2 \\ & 1 & -3 & 1 \\ & & 15 & 3 \\ & & & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 39 \\ -9 \end{bmatrix}$$

$$x_4 = 3$$

$$x_3 = \frac{39 - 3 - 3}{15} = 2$$

$$x_2 = \frac{-2 - 3 - (-3) \cdot 2}{1} = 1$$

$$x_1 = \frac{-1 - (-2) \cdot 3 - 1 \cdot 2 - 1 \cdot 1}{2} = 1$$

$$x = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

## Question 2

$$i = 1 \quad k = 4 \quad [4 \ 2 \ 3 \ 1]$$

$$l_1 = \begin{bmatrix} 1/8 \\ 1/4 \\ 1/2 \\ 1 \end{bmatrix} \quad u_1 = [32 \ 24 \ 10 \ 11]$$

$$A_2 = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 8 & 8 & 5 & 2 \\ 16 & 12 & 10 & 5 \\ 32 & 24 & 20 & 11 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 5/2 & 11/8 \\ 8 & 6 & 5 & 11/4 \\ 16 & 12 & 10 & 11/2 \\ 32 & 24 & 20 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1/2 & -3/8 \\ 0 & 2 & 0 & -3/4 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$i = 2 \quad k = 2 \quad p = [4 \ 2 \ 3 \ 1]$$

$$l_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad u_2 = [0 \ 2 \ 0 \ -3/4]$$

$$A_3 = \begin{bmatrix} 0 & 0 & -1/2 & -3/8 \\ 0 & 2 & 0 & -3/4 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -3/4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1/2 & -3/8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$i = 3 \quad k = 4 \quad p = [4 \ 2 \ 1 \ 3]$$

$$l_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad u_3 = [0 \ 0 \ -1/2 \ -3/8]$$

$$A_4 = \begin{bmatrix} 0 & 0 & -1/2 & -3/8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -1/2 & -3/8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$i = 4 \quad k = 4 \quad p = [4 \ 2 \ 1 \ 3]$$

$$l_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad u_4 = [0 \ 0 \ 0 \ -1/2]$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$L = P * \begin{bmatrix} 1/8 & 0 & 1 & 0 \\ 1/4 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 1 & 0 & 0 \\ 1/8 & 0 & 1 & 0 \\ 1/2 & 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 32 & 24 & 20 & 11 \\ 0 & 2 & 0 & -3/4 \\ 0 & 0 & -1/2 & -3/8 \\ 0 & 0 & 0 & -1/2 \end{bmatrix}$$

## Question 4

A =

1 4 8 3 4 20 40 28 8 40 89 71 3 28 71 114

ans =

1 4 8 3

S =

1 4 8 3 4 16 32 12 8 32 64 24 3 12 24 9

A =

0 0 0 0 4 8 16 0 8 25 47 0 16 47 105

ans =

0 2 4 8

S =

0 0 0 0 4 8 16 0 8 16 32 0 16 32 64

A =

0 0 0 0 0 0 0 0 9 15 0 0 15 41

ans =

0 0 3 5

```
S =  
0 0 0 0 0 0 0 0 0 0 9 15 0 0 15 25  
A =  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 16  
ans =  
0 0 0 4
```