Howework 1 – Introduction to Computational Science

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Question 5

Point a)

$1.1110_2 * 2^4$, so $d_0 = 1$, $d_1 = 1$, $d_2 = 1$, $d_3 = 1$, $d_4 = 0$, and $e = 4$.

Point b)

$1.1110_2 * 2^1$, so $d_0 = 1$, $d_1 = 1$, $d_2 = 1$, $d_3 = 1$, $d_4 = 0$, and $e = 1$.

Point c)

0.00048828125, or $2^{-4} * 2^{-7}$

Point d)

248, or $(2 - 2^{-4}) * 2^7$

Point e)

The best approximation for 30.1 is 30, or $1.1110_2 * 2^4$, so $d_0 = 1$, $d_1 = 1$, $d_2 = 1$, $d_3 = 1$, $d_4 = 0$, and $e = 4$. This representation is not exact. The number 30.1 cannot be represented exactly in this format or any format with $e = 2$.

Question 6

Point a)

$$f(x) = \frac{1}{x} - \frac{1}{x+1} = \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} = \frac{x+1-x}{x(x+1)} = \frac{1}{x(x+1)} = g(x)$$

Point b)

$$f(10_f) = \frac{1}{1.00 * 10^1} - \frac{1}{(1.00 * 10^1) + 1} = (1.00 * 10^{-1}) - \frac{1}{1.10 * 10^1} \approx$$

$$\approx (1.00 * 10^{-1}) - (0.91 * 10^{-1}) = 9.00 * 10^{-3} = 0.009$$

$$g(10_f) = \frac{1}{(1.00 * 10^1)((1.00 * 10^1) + 1)} = \frac{1}{1.00 * 10^1 * 1.10 * 10^1} \approx (9.09 * 10^{-3}) = 0.00909$$

$$f(100_f) = \frac{1}{1.00 * 10^2} - \frac{1}{(1.00 * 10^2) + 1} = (1.00 * 10^{-2}) - \frac{1}{1.01 * 10^2} \approx$$

$$\approx (1.00 * 10^{-2}) - (0.99 * 10^{-2}) = 1.00 * 10^{-4} = 0.0001$$

$$g(100_f) = \frac{1}{(1.00 * 10^2)((1.00 * 10^2) + 1)} = \frac{1}{1.00 * 10^2 * 1.01 * 10^2} \approx (0.99 * 10^{-4}) = 0.000099$$

$$f(1000_f) = \frac{1}{1.00 * 10^3} - \frac{1}{(1.00 * 10^3) + 1} \approx (1.00 * 10^{-3}) - \frac{1}{1.00 * 10^3} =$$

$$= (1.00 * 10^{-3}) - (1.00 * 10^{-3}) = 0$$

$$g(1000_f) = \frac{1}{(1.00 * 10^3)((1.00 * 10^3) + 1)} \approx \frac{1}{1.00 * 10^3 * 1.00 * 10^3} = (1.00 * 10^{-6}) = 0.000001$$

Point c)

Value	Correct	Abs. Error	Rel. error
$f(10_f)$	$0.\overline{90}$	$9.\overline{09} * 10^{-5}$	$8.2645 * 10^{-7}$
$g(10_f)$	$0.\overline{90}$	$9.\overline{09} * 10^{-7}$	$8.26 * 10^{-9}$
$f(100_f)$	$9.\overline{9009} * 10^{-5}$	$9.901 * 10^{-7}$	10^{-2}
$g(100_f)$	$9.\overline{9009} * 10^{-5}$	$9.9 * 10^{-9}$	10^{-4}
$f(1000_f)$	$9.99 * 10^{-7}$	$9.99 * 10^{-7}$	1
$g(1000_f)$	$9.99 * 10^{-7}$	10^{-9}	10^{-3}

Point d)

$f(x)$ is clearly less accuratee than $g(x)$ when dealing with floating point numbers. This is largely due to the final subtraction, since that amplifies approximation errors already introduced in the division $\frac{1}{x+1}$. Also, the multiplication in $\frac{1}{x(x+1)}$ does not cause any approximation errors with the numbers tested, since the mantissa is always 1 and the multiplication of the exponent part is always exact.