Howework 4 – Introduction to Computational Science

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Question 1

$$L_0(x) = \prod_{j=0, j\neq 0}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x-(-0.5)}{(-1)-(-0.5)} \cdot \frac{x - 0.5}{(-1)-0.5} \cdot \frac{x-1}{(-1)-1} = -\frac{2}{3}x^3 + \frac{2}{3}x^2 + \frac{1}{6}x - \frac{1}{6}$$

$$L_1(x) = \prod_{j=0, j\neq 1}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x-(-1)}{(-0.5)-(-1)} \cdot \frac{x - 0.5}{(-0.5)-0.5} \cdot \frac{x-1}{(-0.5)-1} = \frac{4}{3}x^3 - \frac{2}{3}x^2 - \frac{4}{3}x + \frac{2}{3}$$

$$L_2(x) = \prod_{j=0, j\neq 2}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x-(-1)}{0.5-(-1)} \cdot \frac{x-(-0.5)}{0.5-(-0.5)} \cdot \frac{x-1}{0.5-1} = -\frac{4}{3}x^3 - \frac{2}{3}x^2 + \frac{4}{3}x + \frac{2}{3}$$

$$L_3(x) = \prod_{j=0, j\neq 3}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x-(-1)}{1-(-1)} \cdot \frac{x-(-0.5)}{1-(-0.5)} \cdot \frac{x-0.5}{1-0.5} = \frac{2}{3}x^3 + \frac{2}{3}x^2 - \frac{1}{6}x - \frac{1}{6}$$

$$p(x) = \sum_{i=0}^{n} y_i L_i(x) = 2 \cdot \left(-\frac{2}{3}x^3 + \frac{2}{3}x^2 + \frac{1}{6}x - \frac{1}{6}\right) + 1 \cdot \left(\frac{4}{3}x^3 - \frac{2}{3}x^2 - \frac{4}{3}x + \frac{2}{3}\right) +$$

$$0.5 \cdot \left(-\frac{4}{3}x^3 - \frac{2}{3}x^2 + \frac{4}{3}x + \frac{2}{3}\right) + 0.4 \cdot \left(\frac{2}{3}x^3 + \frac{2}{3}x^2 - \frac{1}{6}x - \frac{1}{6}\right) = -\frac{2}{5}x^3 + \frac{3}{5}x^2 - \frac{2}{5}x + \frac{3}{5}$$

$$\frac{\max_{x\in[-1,1]} |f^{(n+1)}|}{5!} = \frac{\max_{x\in[-1,1]} |\frac{7680}{|2x+3|^6}|}{120} = \max_{x\in[-1,1]} \frac{64}{|2x+3|^6} = 64$$

$$\max_{x\in[-1,1]} \left|(x-1)\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x+1)\right| = \max_{x\in[-1,1]} \left|x^4 - \frac{5}{4}x^2 + \frac{1}{4}\right| = \frac{1}{4}$$

$$\max_{x\in[-1,1]} |f(x) - p(x)| \leq \frac{\max_{x\in[-1,1]} |f^{(n+1)}|}{5!} \max_{x\in[-1,1]} \left|(x-1)\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x+1)\right| =$$

$$= 64 \cdot \frac{1}{4} = 8 \leq 8$$

The statement above is true so p satisfies the error estimate:

$$\max_{x\in[-1,1]} |f(x) - p(x)| \leq 8$$

Question 2

We first use the Lagrange method:

$$L_1(x) = \prod_{j=0, j\neq 1}^{2} \frac{x - x_j}{x_i - x_j} = \frac{x-0}{1-0}\frac{x-3}{1-3} = -\frac{1}{2}x^2 + \frac{3}{2}x$$

$$L_2(x) = \prod_{j=0, j\neq 2}^{2} \frac{x - x_j}{x_i - x_j} = \frac{x-0}{3-0}\frac{x-1}{3-1} = \frac{1}{6}x^2 - \frac{1}{6}x$$

$$p(x) = (-3) \cdot \left(-\frac{1}{2}x^2 + \frac{3}{2}x\right) + 1 \cdot \left(\frac{1}{6}x^2 - \frac{1}{6}x\right) = \frac{5}{3}x^2 - \frac{14}{3}x$$

Then we use the Newtonian method:

$$a_0 = f[0] = 0, \qquad f[1] = -3 \qquad f[3] = 1$$

$$a_1 = f[0,1] = \frac{-3-0}{1-0} = -3, \qquad f[1,3] = \frac{1-(-3)}{3-1} = 2$$

$$a_2 = f[0,1,3] = \frac{2-(-3)}{3-0} = \frac{5}{3}$$

$$p(x) = \left(\frac{5}{3}(x-1) - 3\right)x + 0 = \frac{5}{3}x^2 - \frac{14}{3}x$$

The interpolating polynomials are indeed equal.