Howework 4 – Introduction to Computational Science

Claudio Maggioni

May 14, 2020

Question 1

$$L_0(x) = \prod_{j=0,j\neq 0}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x - (-0.5)}{(-1) - (-0.5)} \cdot \frac{x - 0.5}{(-1) - 0.5} \cdot \frac{x - 1}{(-1) - 1} = -\frac{2}{3}x^3 + \frac{2}{3}x^2 + \frac{1}{6}x - \frac{1}{6}$$

$$L_1(x) = \prod_{j=0,j\neq 1}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x - (-1)}{(-0.5) - (-1)} \cdot \frac{x - 0.5}{(-0.5) - 0.5} \cdot \frac{x - 1}{(-0.5) - 1} = \frac{4}{3}x^3 - \frac{2}{3}x^2 - \frac{4}{3}x + \frac{2}{3}$$

$$L_2(x) = \prod_{j=0,j\neq 2}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x - (-1)}{0.5 - (-1)} \cdot \frac{x - (-0.5)}{0.5 - (-0.5)} \cdot \frac{x - 1}{0.5 - 1} = -\frac{4}{3}x^3 - \frac{2}{3}x^2 + \frac{4}{3}x + \frac{2}{3}$$

$$L_3(x) = \prod_{j=0,j\neq 3}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x - (-1)}{1 - (-1)} \cdot \frac{x - (-0.5)}{1 - (-0.5)} \cdot \frac{x - 0.5}{1 - 0.5} = \frac{2}{3}x^3 + \frac{2}{3}x^2 - \frac{1}{6}x - \frac{1}{6}$$

$$p(x) = \sum_{i=0}^{n} y_i L_i(x) = 2 \cdot \left(-\frac{2}{3}x^3 + \frac{2}{3}x^2 + \frac{1}{6}x - \frac{1}{6} \right) + 1 \cdot \left(\frac{4}{3}x^3 - \frac{2}{3}x^2 - \frac{4}{3}x + \frac{2}{3} \right) +$$

$$0.5 \cdot \left(-\frac{4}{3}x^3 - \frac{2}{3}x^2 + \frac{4}{3}x + \frac{2}{3} \right) + 0.4 \cdot \left(\frac{2}{3}x^3 + \frac{2}{3}x^2 - \frac{1}{6}x - \frac{1}{6} \right) = -\frac{2}{5}x^3 + \frac{3}{5}x^2 - \frac{2}{5}x + \frac{3}{5}$$

$$\frac{\max_{x\in[-1,1]} |f^{(n+1)}|}{5!} = \frac{\max_{x\in[-1,1]} \left| \frac{7680}{|2x+3|^6} \right|}{120} = \max_{x\in[-1,1]} \frac{64}{|2x+3|^6} = 64$$

$$\max_{x\in[-1,1]} \left| (x - 1)\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x + 1) \right| = \max_{x\in[-1,1]} \left| x^4 - \frac{5}{4}x^2 + \frac{1}{4} \right| = \frac{1}{4}$$

$$\max_{x\in[-1,1]} |f(x) - p(x)| \le \frac{\max_{x\in[-1,1]} |f^{(n+1)}|}{5!} \max_{x\in[-1,1]} \left| (x - 1)\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x + 1) \right| =$$

$$= 64 \cdot \frac{1}{4} = 8 \le 8$$

The statement above is true so p satisfies the error estimate:

$$\max_{x\in[-1,1]} |f(x) - p(x)| \le 8$$